The state of reliability of the today's complex and costly electronics and photonics systems should be checked (monitored) and, if possible and feasible, even managed on a continuous basis. This is particularly true for systems where failure-free operation is especially important. The objective of technical diagnostics (TD)[1,2] is to recognize, in a continuous fashion and without taking apart the object of interest, its technical state and its ability to continue to perform in the expected (specified) fashion. TD establishes the links between the observed (detected) signals, the so-called "symptoms of faults" (SoF), and the underlying hidden state ("health") of the device or the system of interest. TD effort is naturally focused on the most vulnerable elements (weakest links) of the design and can use the failure-oriented accelerated test (FOAT) data [3] conducted in the design stage.

TD is an important part of reliability engineering and encompasses a broad spectrum of problems associated with obtaining, processing and assessment of diagnostic information, including diagnostic models, decision making rules and algorithms. TD provides information for the subsequent prognostics and health monitoring/management (PHM) effort.[4] TD has to devise solutions and recommendations ("educated guesses") under conditions of uncertainty and with limited information. Therefore the TD methods, techniques and algorithms are based, as a rule, on the probabilistic risk management and applied probability bodies of knowledge and are supposed to quantify, on the probabilistic basis, the obtained information (signals, SoF) and to provide assistance in making a decision if the device or a system of interest is still sound or has become faulty. There is always the possibility that the interpretation of the obtained SoF signal might be a false alarm or might lead to a missing-a-target decision. Statistical theory of decision-making, which is widely employed in radar engineering and is part of the TD, can be effectively used to avoid a false-alarm/missing-a-target mistake.

The objective of the analysis that follows is to show how the statistical Bayes formula (theorem) to update beliefs[5-10] can be used to interpret the TD (PoF) information and to determine if the device (system) of interest is still sound (healthy) or has become faulty, and to use this information to identify a faulty device, if any. Then a reliability physics-oriented Boltzmann-Arrhenius-Zhurkov (BAZ) model [11-13] can be employed to estimate the remaining useful lifetime (RUL) [4,14] of a damaged (faulty) device. When the PDfR concept is used, [2,15] such an assessment will lead, of course, to different RUL predictions, depending on the level of the allowable probability of failure (PoF).

Bayes formula in TD problems. Bayes' theorem to update beliefs is widely used in many areas of applied science, engineering, economics, game theory, medicine and even law. In this section we interpret the Bayes' formula in application to TD problems.

Let an event *S* be the observed (detected) signal (SoF), such as, e.g., measured elevated off-normal temperature, elevated leakage current, drop in the output light intensity, elevated amplitudes (power spectrum) of the induced vibrations, etc., and the events *Di*, *i*

= 1,2,3...be possible malfunctions, diagnosed deviations from the normal operation conditions of the product (system) elements that might be responsible for the observed symptom(s). It is assumed that one and only one of the product elements is damaged to an extent that its detected off-normal performance has manifested itself in the observed symptom. Simultaneous failure (damage) of two or more systems' elements is deemed to be extremely unlikely and is excluded from consideration.

Let one know the typical probabilities *P(Di)* of failure of its particular elements, based on the accumulated experience for the type of the device or system in question. The problem of interest can be formulated this way: The event (signal)
S
is observed for the given device (system) in operation. What is the probability that it is the system's particular i-th element that has become faulty and is therefore responsible for the detected symptom?

The Bayes formula

$$P(D_i/S) = \frac{P(D_i)P(S/D_i)}{\sum_{j=1}^{n} P(D_j)P(S/D_j)},$$

enables one to determine the posteriori probability *P(Di)/S)*, after the symptom *S* has been detected, from the
priori
probability
P(Di)
of the typical, known from the previous experience, probability of the system's state. The Bayes formula can be obtained from the complete probability formula

$$P(S) = \sum_{j=1}^{n} P(D_j)P(S/D_j),$$

and the relationship

$$P(S)P(D_i/S) = P(D_i)P(S/D_i)$$

Formula 2 reflects a postulate that if a system has several possible and incompatible ways to get transferred from the state *Dj* to the state *S*, the probability of such an event can be found as the sum of the conditional probabilities of occurrence of each of these ways. Formula 3 indicates the probability of the simultaneous occurrence of the symptom
S

and the system condition (diagnosis)
Di
.

As follows from Bayes formula,

$$\sum_{i=1}^{n} P(D_i / S) = 1.$$

Bayes method is simple, easy-to-use and effective, and is widely used in many applied problems. Its shortcomings are the large volume of the required input information and "suppression" of seldom diagnoses.

Example 1. Let it be established from experience with the given devices or systems that 90% of the devices do not fail during the designated time of operation, and the symptom *S*, which is the increase in temperature by 20˚C above the normal level, is encountered in 5% of the cases (devices). The probabilities
P(D1)
and
P(D2)
of the sound condition
D1
and the faulty condition
D2
, respectively, in the general population of devices are
P(D1)
= 0.9 and
P(D2)
= 0.1, respectively. The conditional probabilities are
P(S / D1)
= 0.05 and
P(S / D2)
= 0.95. Let us determine the probability that the device, in which the increase in temperature is detected, is sound. The Bayes formula yields

$$P(D_1 / S) = \frac{P(B_1)P(S / D_1)}{P(D_1)P(S / D_1) + P(D_2)P(S / D_2)} = \frac{0.9 \times 0.05}{0.9 \times 0.05 + 0.10 \times 0.95} = 0.32.$$

Thus, the probability that the device is still sound has decreased because of the detected increase in the observed temperature, from 0.90 to 0.32.

Example 2. The TD instrumentation has detected two deviations (SoF) from normal operation conditions: increase in temperature by 20˚C in the region of the heat sink location (symptom

S1
) and increase in the power of the vibration spectrum by 20% (symptom
S2
). It has been established that these symptoms might be due to the possible malfunction of one of the two pieces of hardware: heat sink (state
D1
) and/or vibration damping equipment (state
D2
). It has been established also that the symptom
S1
(increase in temperature) is not observed at normal operation condition (state
D3
), and the symptom
S2
is observed in 5% of cases (devices). Based on the existing experience of employing the devices of interest, it has been established that 80% of the devices do not fail during the specified time of operation; 5% of the devices are characterized by the state
D1
(malfunction of the heat sink), and 15% are characterized by the state
D2
(malfunction of the vibration damping system). It also has been established that the symptom
S1
is encountered in the state
D1
in 20% of the devices, and in the state
D2
in 40% of the devices; that the symptom
D2
is encountered in the state
D1
in 30% of the devices, and in the state
D2
in 50% of the devices. This information can be conveniently presented in the form of a diagnostic matrix
(Table 1)
.

TABLE 1. Diagnostics Matrix

D_i	$P(S_1/D_i)$	$P(S_2/D_i)$	$P(S_3/D_i)$
D_1	0.2	0.3	0.05
D_2	0.4	0.5	0.15
D_3	0	0.05	0.8

Let us determine first the probabilities of the device states, when both symptoms, *S1* and *S2*, have been detected. The Bayes formula
1

yields

$$P(D_1/S_1S_2) = \frac{0.05\text{x}0.20\text{x}0.30}{0.05\text{x}0.20\text{x}030 + 0.15\text{x}0.40\text{x}0.50 + 0.80\text{x}0.00\text{x}0.05} = 0.09.$$

Similarly, we find: *P(D2 /S1S2)* = 0.91; *P(D3/S1S2)* = 0. Determine now the probability of the device state, if the observations indicated that there was no increase in temperature (the symptom *S1* does not take place), but the symptom *S2* (increase in the power spectrum of the induced vibrations) was detected. The absence of the symptom *S1* means that the symptom of the opposite event takes place, so that $P(\bar{S}_1/D_i) = 1 - P(S_1/D_i).$ Changing the probability *P(S1/Di)* in the diagnostics matrix to $P(\bar{S}_1/D_i)$, we find

$$P(D_1/\bar{S}_1S_2) = \frac{0.05\text{x}0.80\text{x}0.30}{0.05\text{x}0.80\text{x}030 + 0.15\text{x}0.60\text{x}0.50 + 0.80\text{x}1.00\text{x}0.05} = 0.12$$

Similarly, we obtain $P(D_2/\bar{S}_1S_2) = 0.46;\ P(D_3/\bar{S}_1S_2) = 0.41.$ Determine now the probabilities of the device states when none of the symptoms takes place. By analogy with the above calculations, we find

$$P(D_1/\bar{S}_1\bar{S}_2) = \frac{0.05\text{x}0.80\text{x}0.70}{0.05\text{x}0.80\text{x}0.70 + 0.15\text{x}0.60\text{x}0.50 + 0.80\text{x}1.00\text{x}0.15} = 0.03.$$

Similarly, we obtain: $P(D_2/\bar{S}_1\bar{S}_2) = 0.05;\ P(D_3/\bar{S}_1\bar{S}_2) = 0.92.$ The calculations indicate that when both symptoms *S1* and *S2* are observed, the state *D1* (the heat sink is malfunctioning) might occur with the probability 0.91. When none of these symptoms is observed, the normal state *D3* is characterized by the probability 0.92 and is the most likely one to occur. When the symptom *S1* (elevated temperature) is not observed, while the symptom *S2* (elevated vibrations) is, the probabilities of the states *D2* (damping system is not working properly) and *D3* (both heat transfer and vibration damping hardware work normally) are 0.45 and 0.41, respectively. Since these probabilities are close, additional information based on observations and/or modeling might be needed to obtain more accurate diagnostics information.

Boltzmann-Arrhenius-Zhurkov's (BAZ) model. Bayes formula (1) does not require any

information about the physical nature of the obtained signals. When there is a reason to believe that the combination of elevated temperature and stress (not necessarily mechanical) can lead to a malfunction of a device or a system, the additional information about the possible source of the deviation of the system's state from the normal operation conditions could be obtained by using Boltzmann-Arrhenius-Zhurkov (BAZ) model
11-13

$$\tau = \tau_0 \exp\left(\frac{U_0 - \gamma\sigma}{kT}\right)$$

that enables one to evaluate the mean time to failure (MTTF) τ from the known applied stress σ (not necessarily mechanical); the absolute temperature
T
, the time constant
$\tau 0$
, the (stress-independent) binding (activation) energy
$U0$
;
k
= 1.3807 x 10-23J/
0
K is Boltzmann's constant, and the factor
γ
is the material (device) constant that is a measure of the vulnerability of the material to the applied stress and is measured by energy per unit stress, so that the product
$\gamma\sigma$
measured in energy units.

BAZ model proceeds from the rationale that although the process of accumulation of damages is temperature dependent, it is influenced primarily by an external loading of any relevant nature. In other words, the model is based on the recognition of the experimentally observed situation that the breakage of the chemical bonds in a material under stress is due primarily to this stress, while temperature plays an important, but not a prevailing, role. Since the BAZ model contains three empirical parameters, activation energy $U0$, parameter γ of the level of the disorientation of the molecular structure of the material, and the time constant
$\tau 0$
, three failure-oriented accelerated test (FOAT) series should be conducted to determine these parameters.

Let FOATs characterized by their absolute temperatures $T1$, $T2$ and $T3$ and applied stresses σ
1
,

$\sigma 2$
and

σ3
be run until failures, and the respective measured times-to-failure (TTF) be
t1
,
t2
and
t3
, respectively. Based on the observed percentages of failed devices, the probabilities of failure (PoF) where established are
Q1
,
Q2
and
Q3
respectively. Assuming that the BAZ model and the exponential law of reliability are applicable, the PoF can be defined as

$$Q = 1 - \exp\left(-\frac{t}{\tau}\right) = \exp\left[-\frac{t}{\tau_0}\exp\left(-\frac{U-\gamma\sigma}{kT}\right)\right],$$

where *t* is time in operation. The time constant *t0* in the BAZ model can be found from the transcendental equation

$$\frac{1-\eta_{12}(\tau_0)}{1-\eta_{13}(\tau_0)} = \frac{1-n_{12}}{1-n_{13}},$$

where the following notation is used:

$$\eta_{12}(\tau_0) = \frac{\tau_2}{\tau_1}\frac{\ln\left[-\frac{\tau_0}{t_2}\ln(1-Q_2)\right]}{\ln\left[-\frac{\tau_0}{t_1}\ln(1-Q_1)\right]}, \quad \eta_{13}(\tau_0) = 1-\frac{\tau_3}{\tau_1}\frac{\ln\left[-\frac{\tau_0}{t_3}\ln(1-Q_3)\right]}{\ln\left[-\frac{\tau_0}{t_1}\ln(1-Q_1)\right]}, \; n_{12} = \frac{\sigma_2}{\sigma_1}, n_{13} = \frac{\sigma_3}{\sigma_1}$$

Then (stress-free) activation energy *U0* and the factor *γ* of loading (power) in the BAZ model can be computed as

Written by Ephraim Suhir, Laurent Bechou and Alain Bensoussan
Monday, 03 December 2012 14:37 - Last Updated Wednesday, 05 December 2012 20:47

$$U_a = \frac{kT_1}{1-n_{12}}\ln\left[-\frac{\tau_2}{t_1}\ln(1-Q_1)\right]\ln\{-\eta_{12}(\tau_2)\} = \frac{kT_1}{1-n_{13}}\ln\left[-\frac{\tau_3}{t_1}\ln(1-Q_1)\right]\ln\{-\eta_{13}(\tau_3)\}$$

$$\tau = \frac{kT_1}{1-n_{12}}\ln\left[-\frac{\tau_2}{t_1}\ln(1-Q_1)\right]\ln\{-\eta_{12}(\tau_2)\} = \frac{kT_1}{1-n_{13}}\ln\left[-\frac{\tau_3}{t_1}\ln(1-Q_1)\right]\ln\{-\eta_{13}(\tau_3)\}$$

$$\tau_0 = 10^{-3}\sec;\ \frac{U_a}{k} = 28350^0 K;$$

$$\frac{\sigma}{k} = 3.0x10^{22} K;\ \gamma = 4712x10^{-23};$$

$$\frac{\gamma\sigma}{k} = 1.0136^0 K;$$

$$\tau = \tau_0 \exp\left(\frac{U_a - \gamma\sigma}{kT}\right) = 10^{-3}\exp[(74.0209 - 35.0769)] = 1.0787x10^5 \sec = 29.9^h$$

$$MTTF = \tau\ln(1-Q)$$

$$\tau = \tau_0 \exp\left(\frac{U_a - \gamma\sigma}{kT}\right) = 10^{-3}\exp(74.0209 - 45.7909) = 0.0167x10^5 \sec = 9.4^h$$

$$\tau = \tau_0 \exp\left(\frac{U_a - \gamma\sigma}{kT}\right) = 10^{-3}\exp(74.0209 - 36.9086) = 1.3112x10^5 \sec = 41.5770$$

$$MTTF = -\tau\ln(1-Q) = -6.5912x10^{-3}\ln(1-10^{-3}) = 0.0065912^h$$

Example 3. Let, e.g., the FQAT carried out until half of the population fails (Q2 = Q3 = Q5) indicate that

so that

then, for the operation temperature of T = 110˚C = 383˚K, the BAZ formula yields:

years. The MTTF will decrease to

years. For the 20˚C increase in temperature, and will be only

[redacted]

Assuming, e.g., $Q = 10^{-3}$, we conclude that although the BUL is only

[redacted]

suhire@aol.com